Chapter 2 Kinematics of particles and rigid bodies
(2-5) 2.4 Kinematics of planar mechanisms

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First Semester,2014-2015

2.4 Kinematics of planar mechanisms (performed with dynamic analysis in Chapter 4)

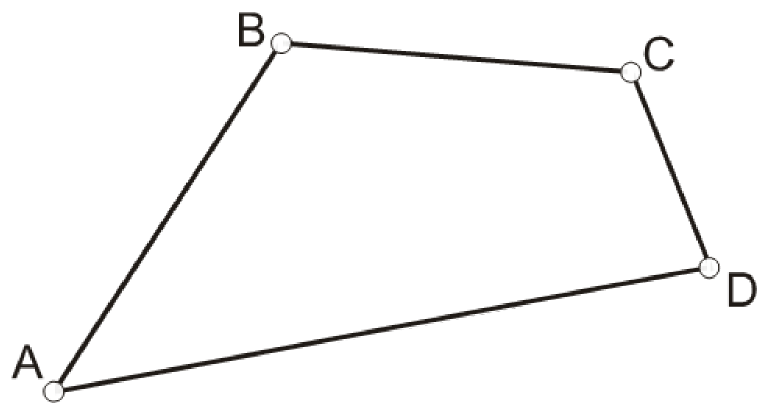
- A free four-bar linkage
- This system has 12 d.o.f. and 8 degrees of constraints, i.e. three d.o.f. as a whole rigid body and one internal d.o.f. (“deformability” of the system).

- Four-bar linkage without any fixed body, with complete freedoms;
- Every rigid body:

  1) translation motion freedom along x axis + translation motion freedom along y axis +revolution freedom around z axis=3 dof/body;

  Further, 3 dof/body*4 body=12 dof /system;

  2) 8 constraints=(revolution constraint around x axis +revolution constraint around y axis)*4;
- Internal dof belongs to the system, is to generate motion then deform the system; is only another expression;



Free four-bar linkage, without frame

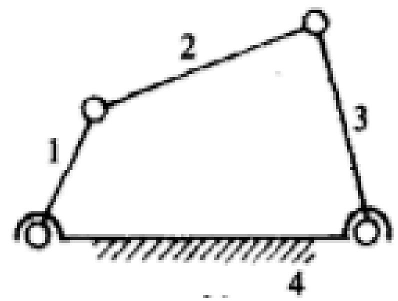
To fix a body, the free four-bar linkage is left with 1 residual dof.

2D)

$$l = 3(m-1) - 2c_1 - c_2$$

m = number of links

c_i = number of joints leaving *i* DOFs free



Four-bar linkage

l = 1

$$l = 3(m - 1) - 2c_1 - c_2$$

$$m = 4$$

$$c_1 = 1 + 1 + 1 + 1 = 4$$

$$c_2 = 0$$

therefore

$$l = 3(4 - 1) - 2?\ 4 \quad 1?\ 0 \quad 1$$

Clockwise analysis : To calculate c_1

- Joint 1-2: between link 1 and 2, only revolution around shaft ,leaving 1 DOF free, then count as 1;
- Joint 2-3: between link 2 and 3, only revolution, leaving 1 DOF free, then count as 1;
- Joint 3-4: between link 3 and 4, only revolution, leaving 1 DOF free, then count as 1;
- Joint 4-1: between link 4 and 1, only revolution, leaving 1 DOF free, then count as 1;

Collection of planar mechanisms

- A collection of rigid bodies properly constrained can be used to create a mechanism.

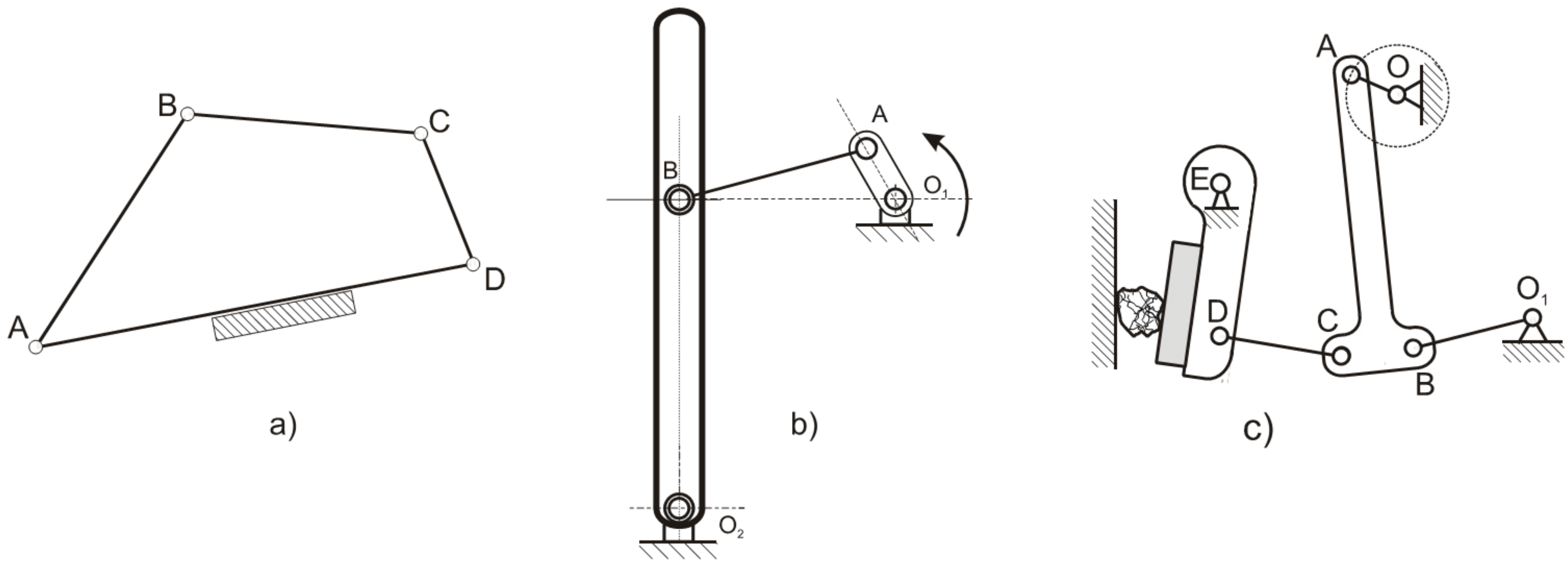
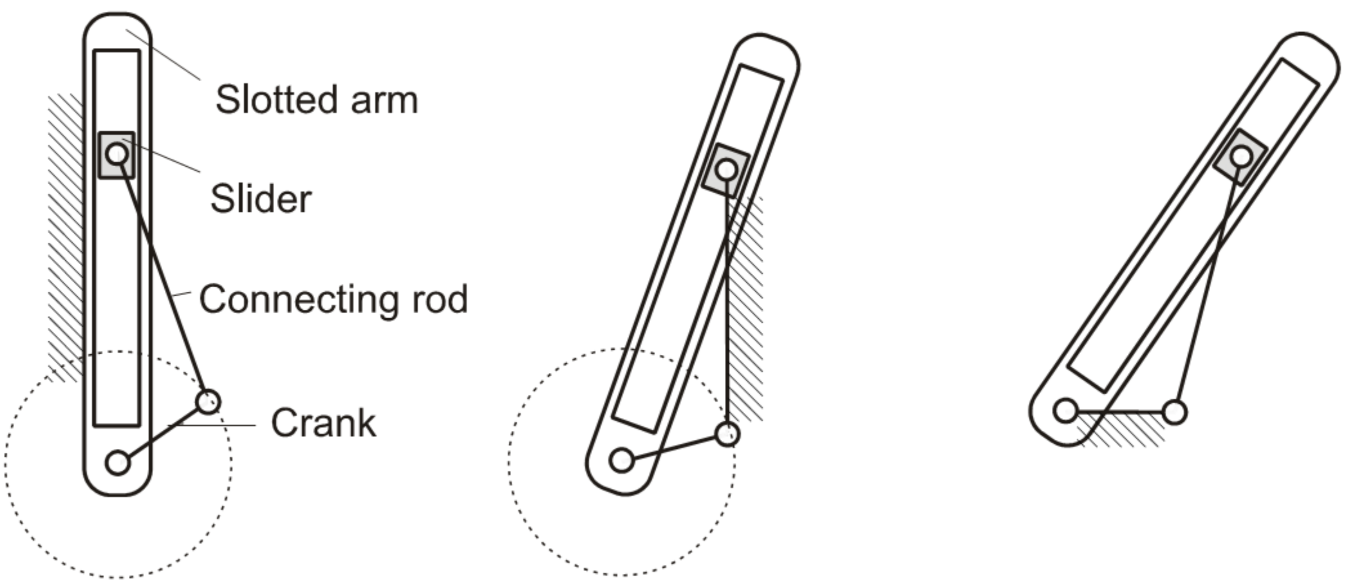


Fig. 2.24

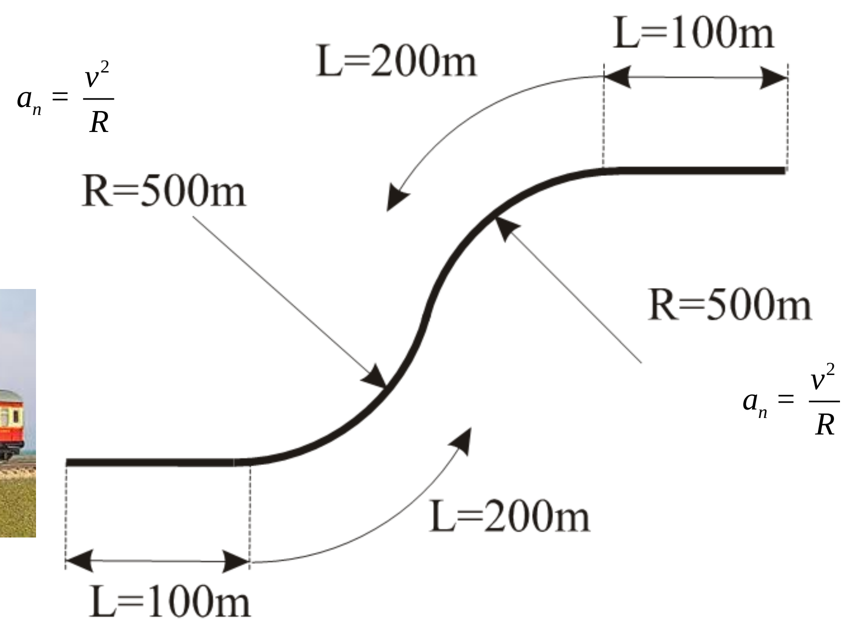
Slider-slotted arm mechanism

- Four rigid bodies: a slotted arm, a slotted arm, a crank, a connecting rod and a slider ;
- Overall 12 dof: same with four-bar linkage;
- 4 double constraints:8 constraints=(revolution constraint around x axis +revolution constraint around y axis)*4; caused by 3 pins and 1 joke(slider);
- To constrain 1 body, then 1 dof left, some diffused mechanisms



2.5 Kinematics of a particle and of rigid bodies: applications

- 1) Motion of a particle on a curvilinear path
- A railway coach moves along the track indicated in the figure below at a constant speed of 20 m/s.
- By considering the coach as a particle, **determine the acceleration** to which the passengers are submitted in the various parts of the track.



Principle : circular motion

- Particle P moves along a circular trajectory, with center O, radius R;
- Assume complex coordinate system coincident with center O;
- Simple to particle P position in polar coordinates;

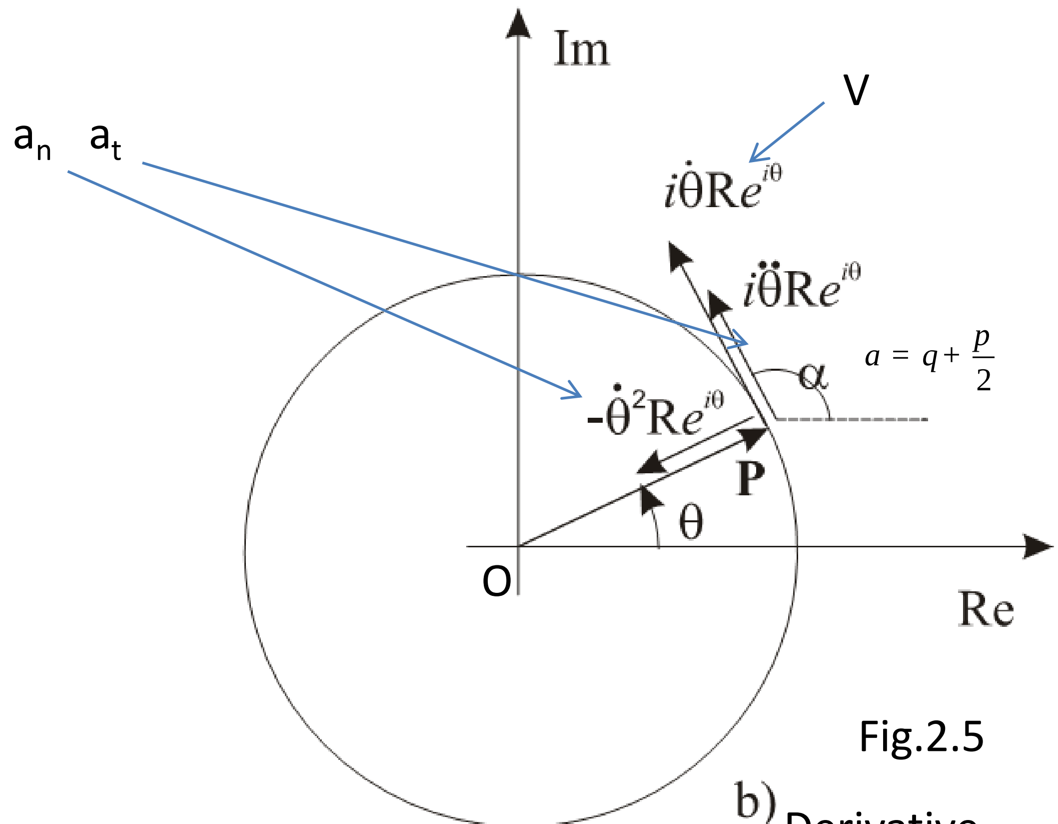


Fig.2.5

b)

$$\overline{P} = R\cos\theta + iR\sin\theta = Re^{i\theta}$$

$$\overline{V} = i\dot{\theta} R e^{i\theta}$$

$$\overline{a} = i\ddot{\theta} R e^{i\theta} - \dot{\theta}^2 R e^{i\theta}$$

Derivative direction, 0 order, 1 order, 2 order of position

Calculation

- Some correction: add a line between s_2 and s_4, and assume tangent with s_2 and s_4;
- Determine the acceleration in parts: s_1 to s_5

$In\ s_1, acceleration\ a_{s1} = 0, as\ velocity\ v_{s1} = 20m/s\ and\ non\ direction\ change;$

$In\ s_2, acceleration\ a_{s2} = -\dot{q}^2 Re^{iq} = -\frac{v^2}{R}e^{iq}, the\ mangitude\ \frac{v^2}{R} = \frac{20^2}{500} = 0.8m/s^2,$

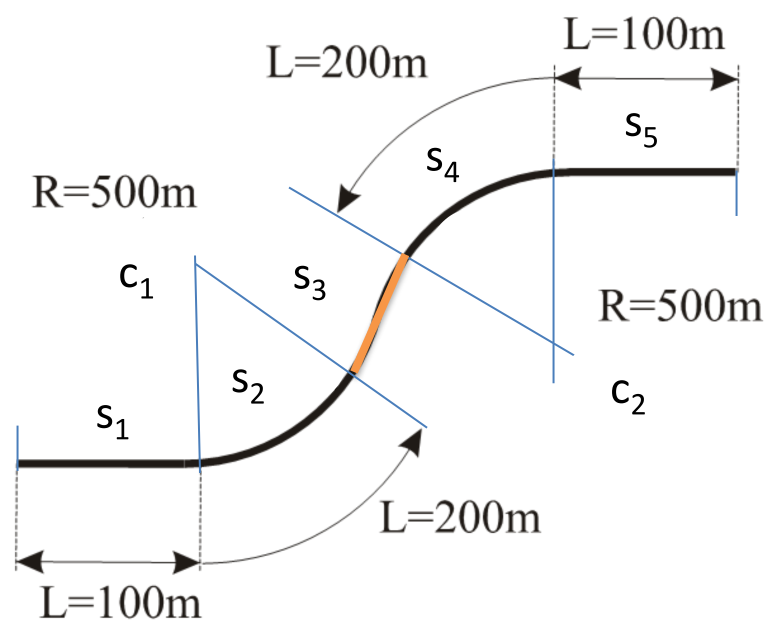
$accleration\ direction\ is\ to\ c_1, as\ velocity\ v_{s2} = 20m/s\ and\ direction\ changes\ along\ circle\ c_1;$

$In\ s_3, acceleration\ a_{s3} = 0, as\ velocity\ v_{s3} = 20m/s\ and\ non\ direction\ change;$

$In\ s_4, acceleration\ a_{s4} = -\dot{q}^2 Re^{iq} = -\frac{v^2}{R}e^{iq}, the\ mangitude\ \frac{v^2}{R} = \frac{20^2}{500} = 0.8m/s^2,$

$accleration\ direction\ is\ to\ c_2, as\ velocity\ v_{s4} = 20m/s\ and\ direction\ changes\ along\ circle\ c_2;$

$In\ s_5, acceleration\ a_{s5} = 0, as\ velocity\ v_{s5} = 20m/s\ and\ non\ direction\ change;$



2) Law of motion

- The load of a **lifting plant** has to be lifted of 24 m starting from idle condition and ending its run at null velocity, with a trapezoidal law of motion.
- The maximum lifting speed is 10 m/min, while the transient motions at the beginning and at the end of the run last 2 s respectively.
- The diagrams of velocity and acceleration are requested, together with the necessary run time.

Calculation

- Vc=10 m/min=1/6 m/s

$$a_1 = Vc/2s = 1/6/2 = 1/12 \quad m/s^2$$

$$a_2 = 0;$$

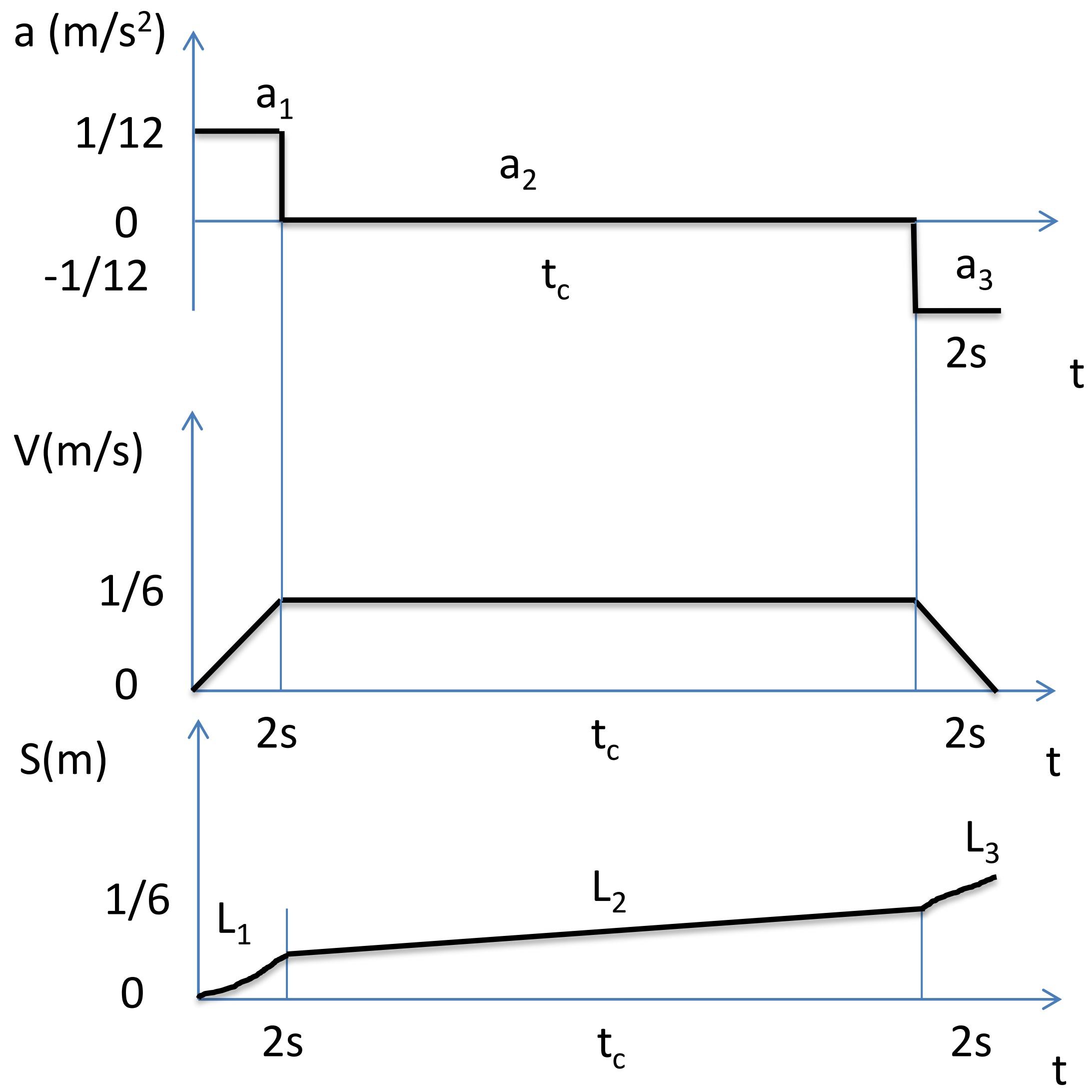
$$a_3 = -Vc/2s = -1/6/2 = -1/12 \quad m/s^2$$

$$L = L_1 + L_2 + L_3 = \int_0^2 a_1 t\,dt + V_c t_c + \int_0^2 |a_2|\,t\,dt$$

$$24 = a_1 \frac{1}{2} t^2 |_0^2 + V_c t_c + |a_2| \frac{1}{2} t^2 |_0^2 = \frac{1}{12}\frac{1}{2} t^2 |_0^2 + \frac{1}{6} t_c + |-\frac{1}{12}| \frac{1}{2} t^2 |_0^2$$

$$24 = \frac{1}{3} + \frac{1}{6} t_c$$

$$t_c = (24 - \frac{1}{3}) * 6 = 142\,s$$

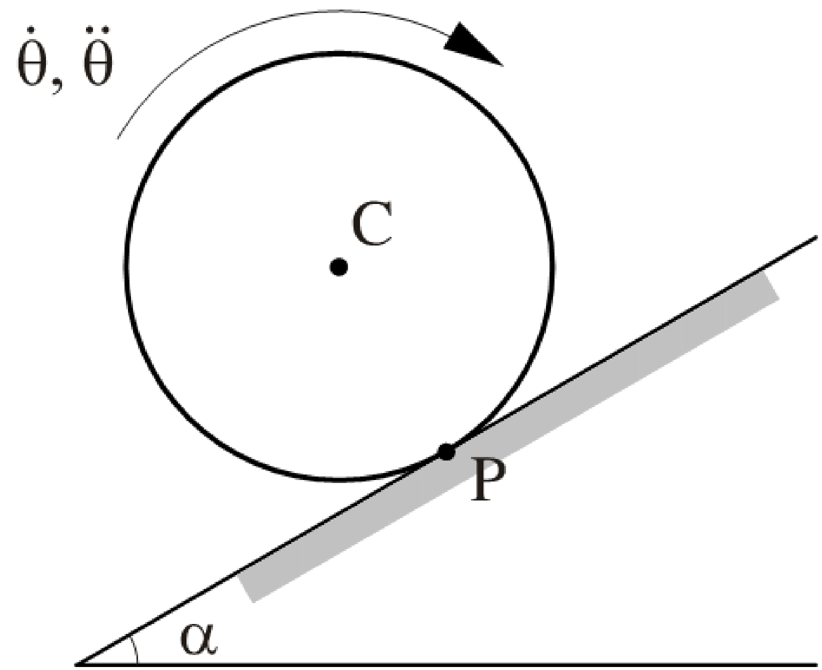


3) Rolling without slipping

- A disc of radius R rolls without slipping on an inclined plane (see figure below).

Given its angular velocity $\dot{\theta}$ and its angular acceleration $\ddot{\theta}$ to determine.

1. the trajectory of the center C of the disc;
2. the instantaneous center of velocity of the disc;
3. velocity and acceleration of the center C of the disc;
4. the acceleration of point P of the disc contacting the plane



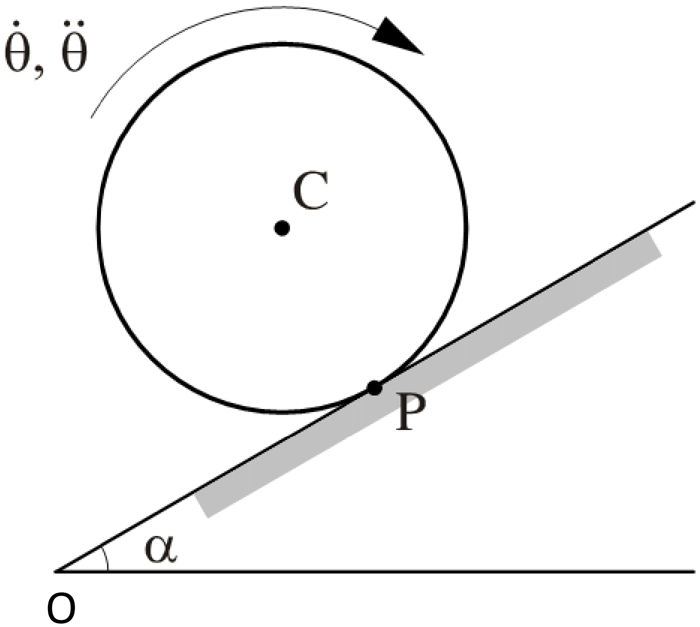
Calculation

1. the trajectory of the center C of the disc;

$$p_c(t) = p_p(t) + (P - C) = r\,\mathrm{e}^{ia} + R\,\mathrm{e}^{i(a+\frac{p}{2})} = \grave{o}\, R\dot{q}t\mathrm{e}^{ia}\,dt + R\,\mathrm{e}^{i(a+\frac{p}{2})}$$

2. the instantaneous center of velocity of the disc;

point P ,as P is the point of the disc has a null velocity in considered instant time;



3. velocity and acceleration of the center C of the disc;

$$V_c(t) = v\,\mathrm{e}^{ia} = R\dot{q}t\mathrm{e}^{ia};$$

$$a_c(t) = a\,\mathrm{e}^{ia} = R\ddot{q}t\mathrm{e}^{ia};$$

4. the acceleration of point P of the disc contacting the plane

$$V_p(t) = 0, \textit{as ponit P contacts with inclined plane}$$

$$a_p(t) = a_c + \ddot{q}?\,(P \quad C) - (\dot{q})^2?\,(P \quad C) = R\ddot{q}t\,\mathrm{e}^{ia} + \ddot{q}?\,\mathrm{Re}^{i(a+\frac{p}{2})} \quad (\dot{q})^2?\,\mathrm{Re}^{i(a+\frac{p}{2})};$$