Chapter 2 Kinematics of particles and rigid bodies
(2-3) Kinematics of rigid body

Bachelor Program in AUTOMATION ENGINEERING

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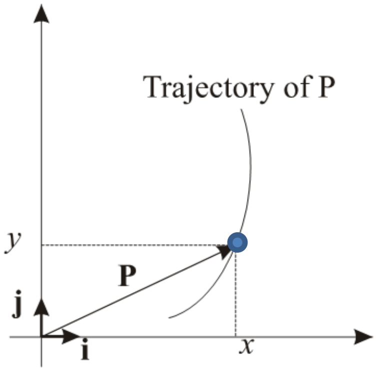
Content of chapter 2

- 1 Mechanisms and machines: Basic Concepts
- 2 Kinematics of particles
- ***3 Kinematics of rigid bodies***

2.2 Kinematics of a rigid body in plane motion

- A particle with a negligible dimension, only as a point;
- A rigid body with finite dimensions, as the real and simplified shape;
- Plane motions of rigid body is considered;



Particle P as a point

The position of a rigid body is determined by the position of its center of mass and by its attitude

What is a rigid body

- Basically, body can be defined as rigid if the distance between any pair of its points remains constant.
- In physics, a **rigid body** is an idealization of a solid body in which deformation is neglected.
- In other words, the distance between any two given points of a rigid body remains constant in time regardless of external forces exerted on it.
- Even though such an object cannot physically exist due to relativity, objects can normally be assumed to be perfectly rigid if they are not moving near the speed of light.
- In classical mechanics a rigid body is usually considered as a continuous mass distribution.

2.2.1 Definition of kinematic quantities

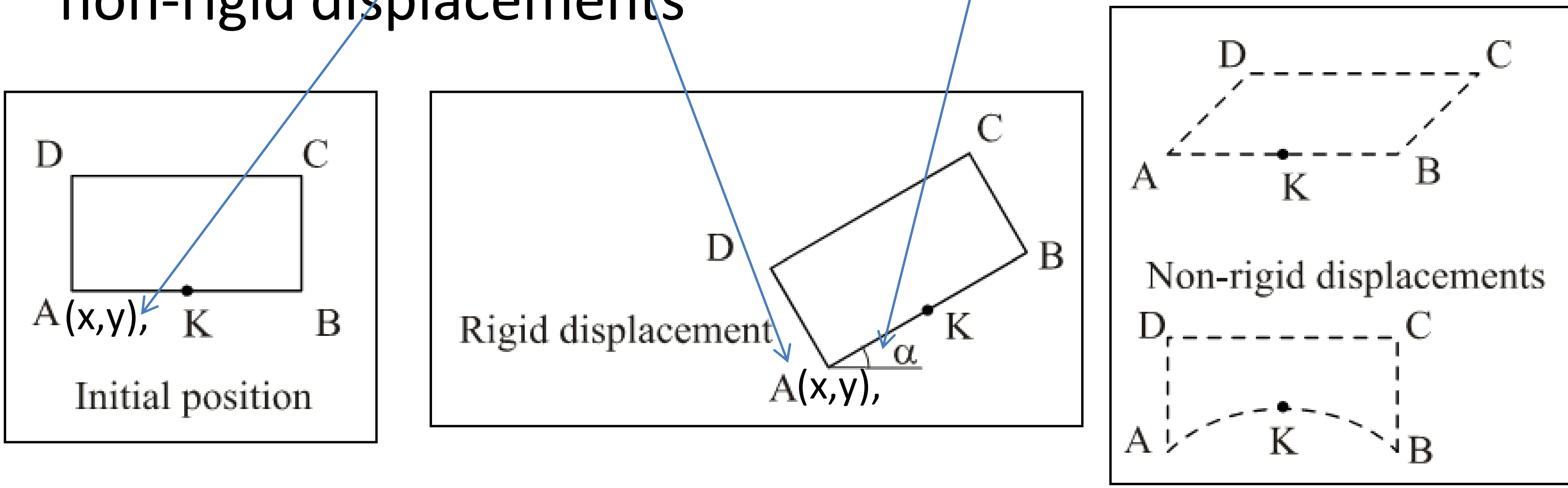
- A body is composed by a certain number of particles;
- The rigid body position: the set of particle position vectors;
- The rigid body motion: particle position vectors variation with time;
- Plane motion: position , velocity, acceleration vectors and trajectories are always parallel to a plane, called directory plane;

2.2.2 Rigid body motions

- In a plane, to location a rigid body, only three coordinates are needed:

two Cartesian coordinates(x,y), + angular coordinate.

- Any combined variation of three coordinates with time leads to rigid motions;
- Three categories: initial position, rigid displacement, non-rigid displacements



2.2.3 Plane rigid motions

- **1, Translation:** The body moves keeping its orientation constant;
- all the points of the body present the same velocities and accelerations.
- The trajectories are rectilinear or curve.

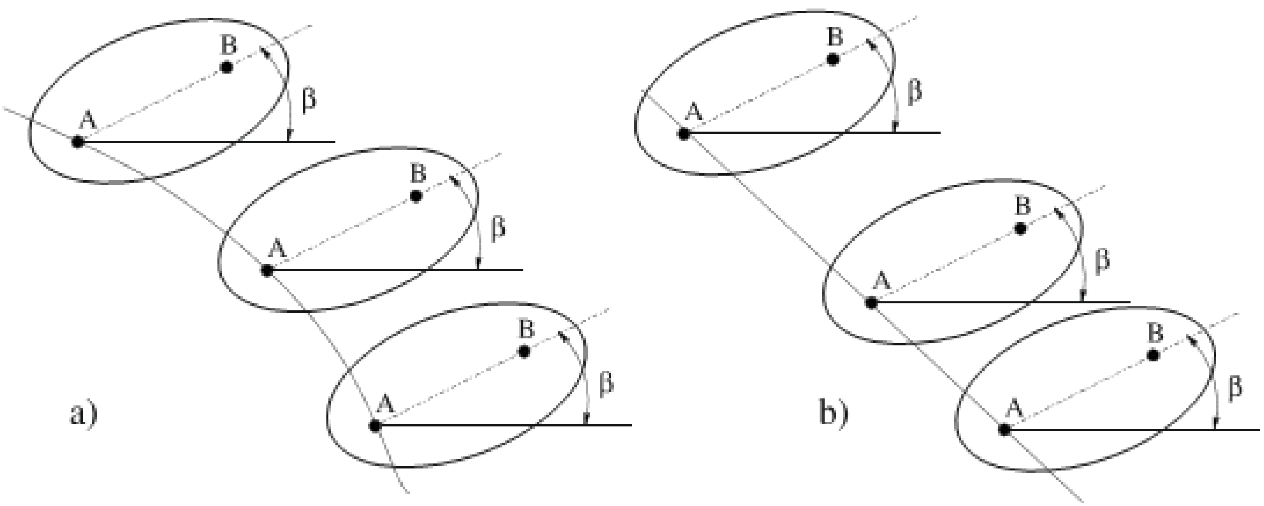


Fig. 2.12

Translation example

APPLICATIONS

Passengers on this amusement ride are subjected to curvilinear translation since the vehicle moves in a circular path but they always remains upright.

2.2.3 Plane rigid motions

- **2, Rotation:** The body moves keeping constant the position of one of its points;
- all other points trajectories are circular.
- The trajectories are circular.

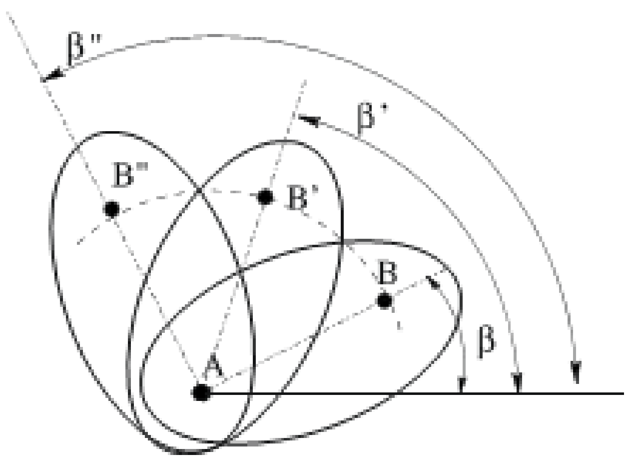
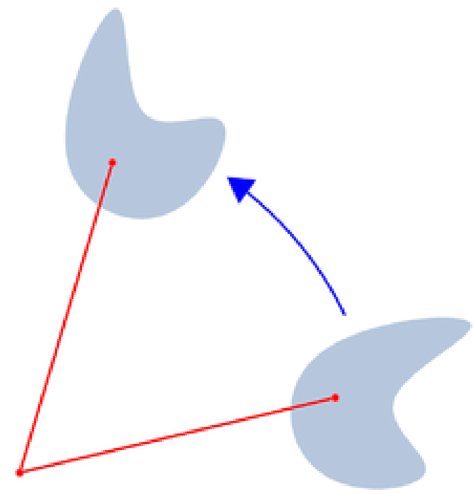


Fig. 2.13

Rotation of a planar figure around a inside point



Rotation of a planar figure around a outside point

Rotation example

Gears rotation

2.2.3 Plane rigid motions

- **3, Translation and Rotation:** The body moves with rotating itself simultaneously;
- All body points are not fixed;
- All rigid body movements are rotations, translations, or combinations of the two.

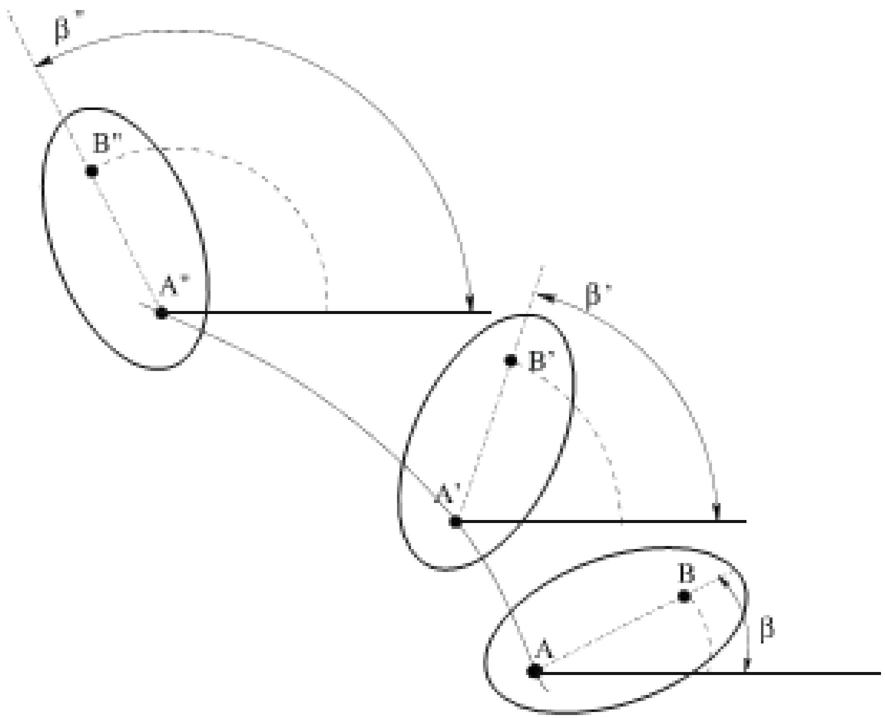
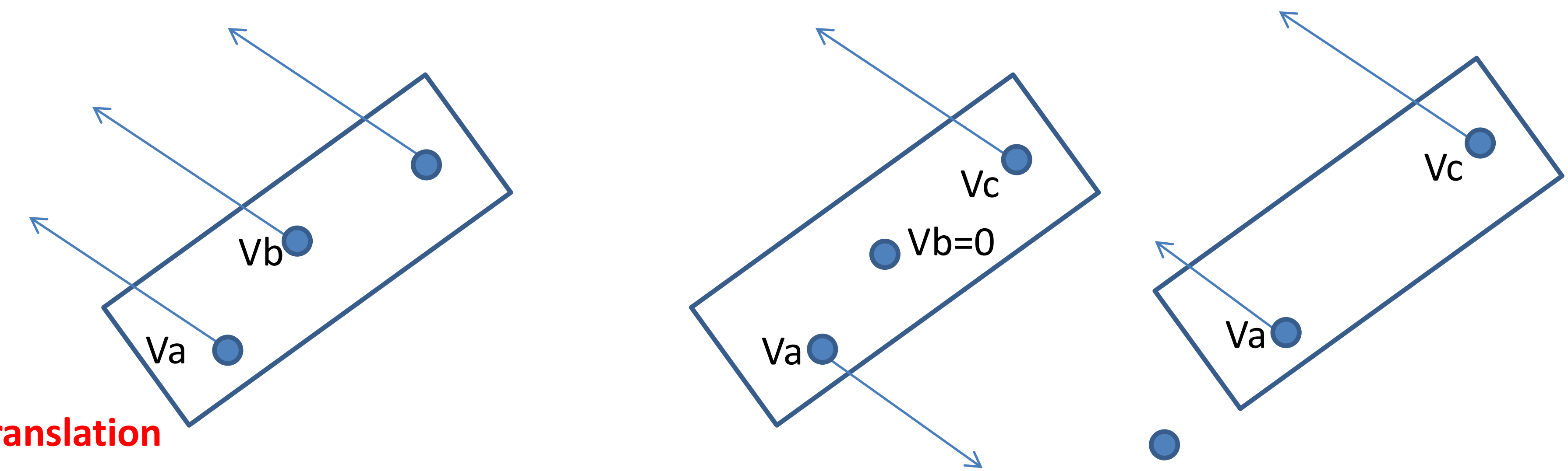


Fig. 2.14

2.2.4 Instantaneous rigid motions

- To take an instantaneous picture of the distribution of the **velocities** of the points of a rigid body in plane motion, an extremely important property holds true: **the instantaneous motion can only be translation or rotation**.



Instantaneous center of velocity (i.c.v.)

Discussion

- Case 1: i.c.v. changing from instant to instant, then the i.c.v. has null velocity but its acceleration is not zero;
- Case 2: i.c.v. acceleration is zero, then the motion is pure rotational;

2.2.5 Velocity and acceleration distributions

- the most general condition of rotation and translation motion by using the coordinate pair x_A, y_A of a point of the body, together with the rotation angle β of the body itself.

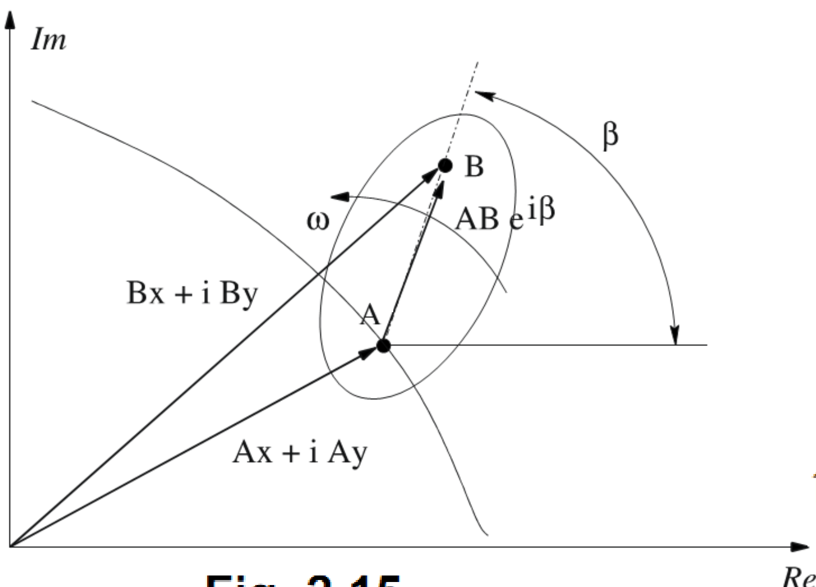


Fig. 2.15

Position

$$\overline{B} = x_B + iy_B = x_A + iy_A + ABe^{i\beta}$$

in polar notation

the vector $ABe^{i\beta}$ from A to B.

2.2.5 Velocity and acceleration distributions

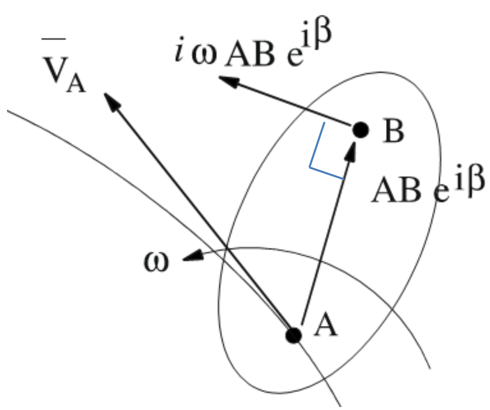
- To calculate the velocity by deriving position

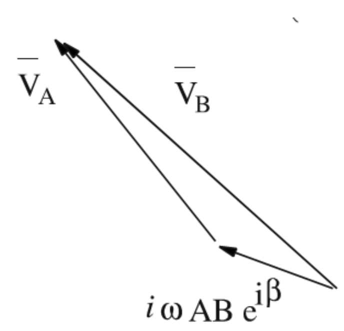
$$\bar{B} = x_B + iy_B = x_A + iy_A + ABe^{i\beta}$$

$$\bar{V}_B = V_{Ax} + iV_{Ay} + i\dot{\beta}ABe^{i\beta} = \bar{V}_A + \dot{\beta}ABe^{i\left(\beta+\frac{\pi}{2}\right)} = \bar{V}_A + \bar{V}_{BA}$$

velocity of an arbitrarily chosen point A

a component perpendicular to line passing through A and B



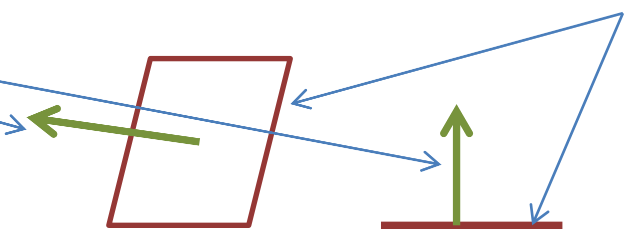


2.2.5 Velocity and acceleration distributions

- To represent the angular velocity around point A

$$\mathbf{V}_B = \mathbf{V}_A + \boldsymbol{\omega} \times (B - A)$$

$\boldsymbol{\omega} = \dot{\beta}\mathbf{k}$, where $\mathbf{k}$ is a unit vector perpendicular to the directory plane

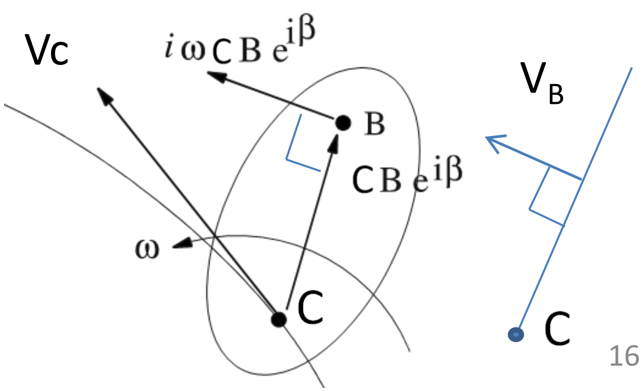


In 3 dimension(x,y,z)， along z axis direction

if instead of a generic point A we use the instantaneous center of velocity C, the velocity of point B becomes:

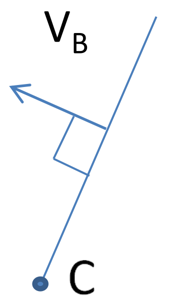
$$\mathbf{V}_B = \boldsymbol{\omega} \times (B - C)$$

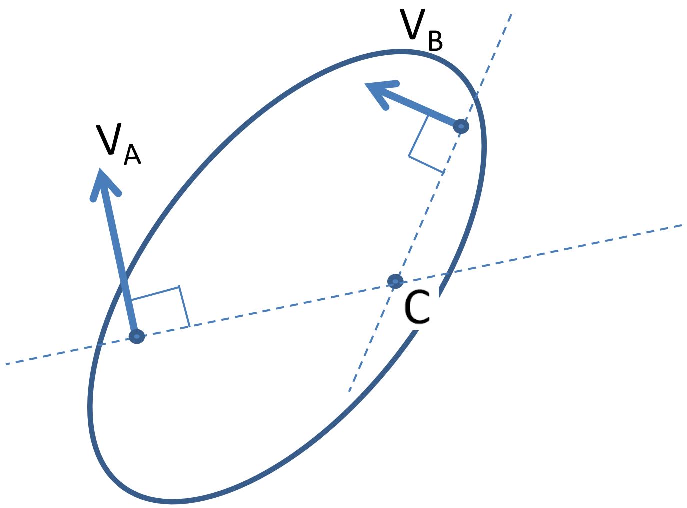
velocity of any point B is **normal** to the line connecting the point to the instantaneous center of rotation C



2.2.5 Velocity and acceleration distributions

- A useful graphic method to determine the instantaneous center of rotation C:
- The intersection point of normal lines of at lest two arbitrary points velocity





2.2.5 Velocity and acceleration distributions

- To calculate acceleration by deriving Velocity:

$$\overline{V}_B = V_{Ax} + iV_{Ay} + i\dot{\beta}ABe^{i\beta} = \overline{V}_A + \dot{\beta}ABe^{i\left(\beta+\frac{\pi}{2}\right)} = \overline{V}_A + \overline{V}_{BA}$$

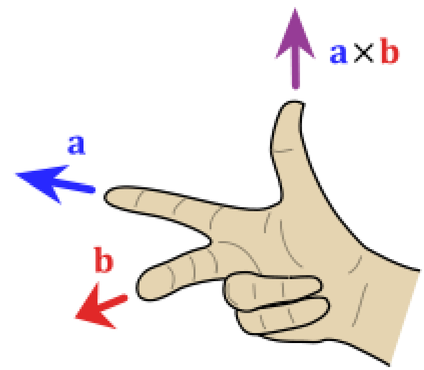
$$\overline{a}_B = a_{Ax} + ia_{Ay} + i\ddot{\beta}ABe^{i\beta} - \dot{\beta}^2ABe^{i\beta}$$

$$\overline{a}_B = a_{Ax} + ia_{Ay} + \ddot{\beta}ABe^{i\left(\beta+\frac{\pi}{2}\right)} - \dot{\beta}^2ABe^{i\beta}$$

in vector form,

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\omega}}\times\left(B-A\right) + \boldsymbol{\omega}\times\left(\boldsymbol{\omega}\times\left(B-A\right)\right)$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\omega}}\times\left(B-A\right) - \omega^2\left(B-A\right) \qquad (2.31)$$



Cross product

2.2.5 Velocity and acceleration distributions

- To simplify by cross product:

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\omega}} \times (B - A) + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times (B - A))$$

(the zero vector).

$$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$$

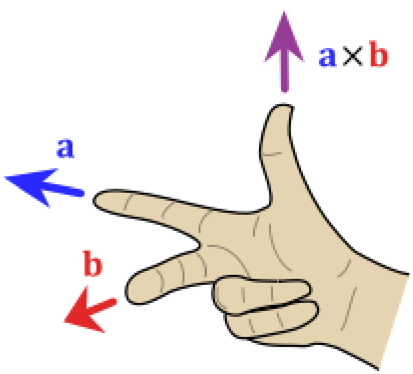
$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\omega}} \times (B - A) - \omega^2 (B - A)$$

$$\boldsymbol{\omega} \times (\boldsymbol{\omega} \times (B - A))$$

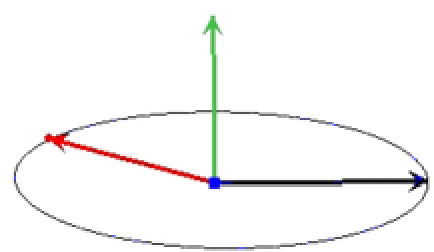
vector

$$\omega^2 (B - A)$$

scalar



Finding the direction of the cross product by the right-hand rule



The cross product (vertical) changes as the angle between the vectors changes

2.2.6 Constraints

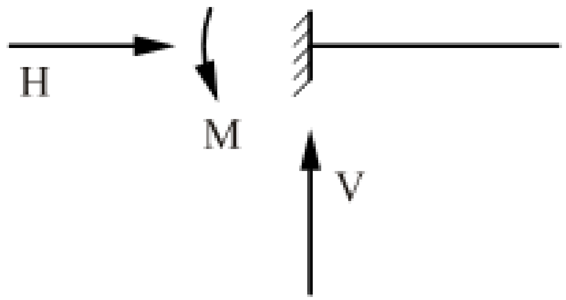
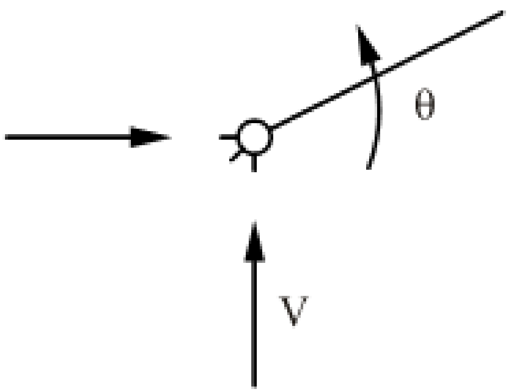
- The degrees of freedom (d.o.f.) of mechanical systems can be limited by the introduction of constraints;
- Represented by equations prescribing displacements and/or rotations to the system components.
- Constraint equations are always given a graphical representation by means of conventional graphic symbols.

2.2.6 Constraints

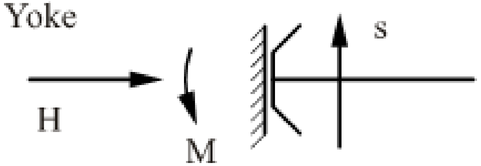
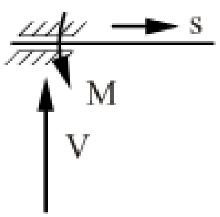
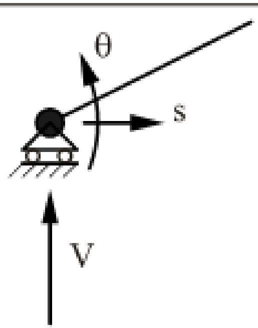
- Constraint reactions: each eliminated d.o.f. there corresponds a force and/or moment to be applied to the mechanical system;
- a constraint reaction is a force if the suppressed d.o.f. is a displacement;
- a constraint reaction is a moment if the suppressed d.o.f. is a rotation.

2.2.6 Constraints

- the graphical symbols of constraints most frequently used

Constraint graphical representation	Constraint Reactions	Remaining d.o.f.
Clamp 	H, V, M	None
Pin 	H, V	θ

2.2.6 Constraints

Yoke 	H, M	s
Sleeve 	V, M	s
Slider 	V	s,θ

Collection of control yokes at Boeing Future of Flight Museum: 747, 707, B-29, Trimotor.

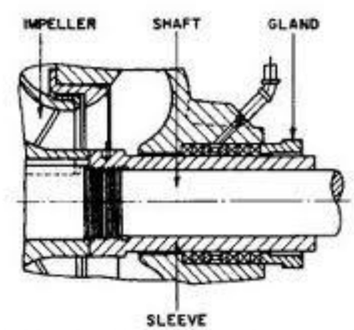


FIGURE 69 A sleeve threaded onto a shaft with no external locknut

2.2.6 Constraints

Contact between non-conforming surfaces

- two solid bodies contact:

conforming contact:

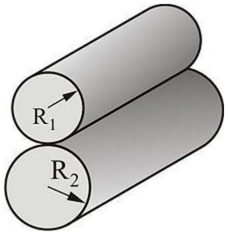
the two bodies touch at multiple points before any deformation takes place (i.e., they just "fit together")

Slider bearing

jonrnal bearing

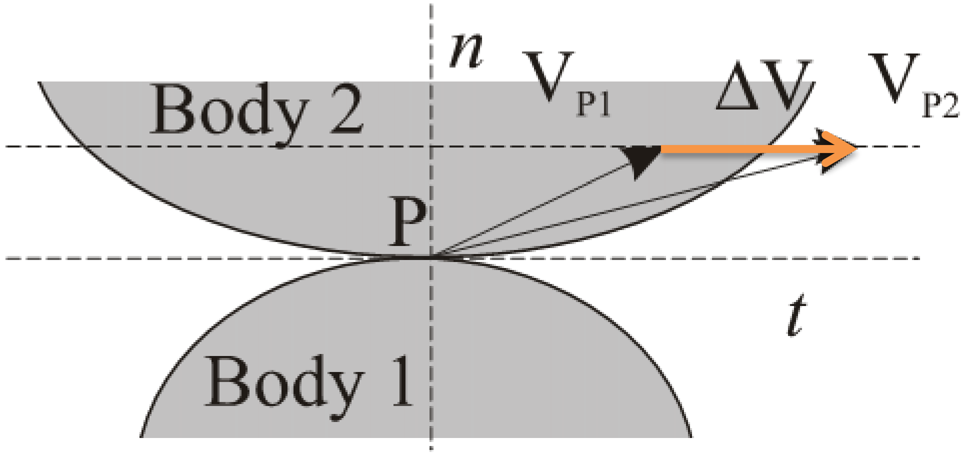
non-conforming contact

the shapes of the bodies are dissimilar enough that, under zero load, they only touch at a point (or possibly along a line)



2.2.6 Constraints

- Contact between non-conforming surfaces:



the slipping speed of the two contacting bodies.

the relative velocity $\Delta \mathbf{V} = \mathbf{V}_{P2} - \mathbf{V}_{P1}$

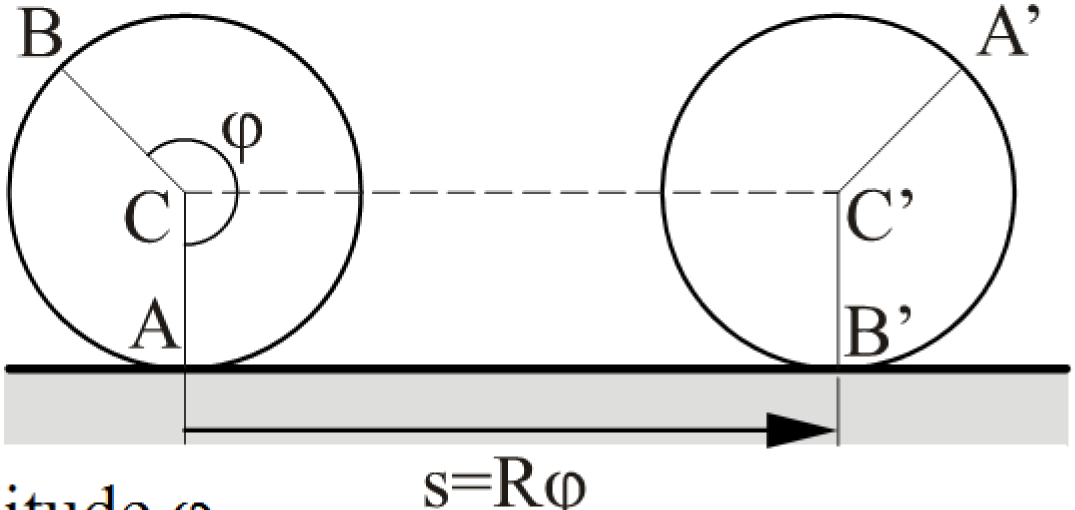
plays a fundamental role in friction and wear phenomena

2.2.6 Constraints

Pure rolling contact (without slipping):

the relative velocity $\Delta \mathbf{V} = \mathbf{V}_{P2} - \mathbf{V}_{P1} = 0$

A disk rolling along a guide



an arc of amplitude φ

length of the arc AB=rφ

$$\frac{ds}{dt} = V_C = R\dot{\varphi}; \qquad \frac{dV_C}{dt} = a_C = R\ddot{\varphi}$$

2.2.6 Constraints

Pure rolling contact :

a pair of contacting rigid bodies both rotating

two circular profiles are rotating in opposite directions and are held in contact in P

the pure rolling constraint: $\varphi_1 R_1 = \varphi_2 R_2$

further with respect to time:

$$\dot{\varphi}_1 R_1 = \dot{\varphi}_2 R_2$$

angular speeds of the two contacting discs

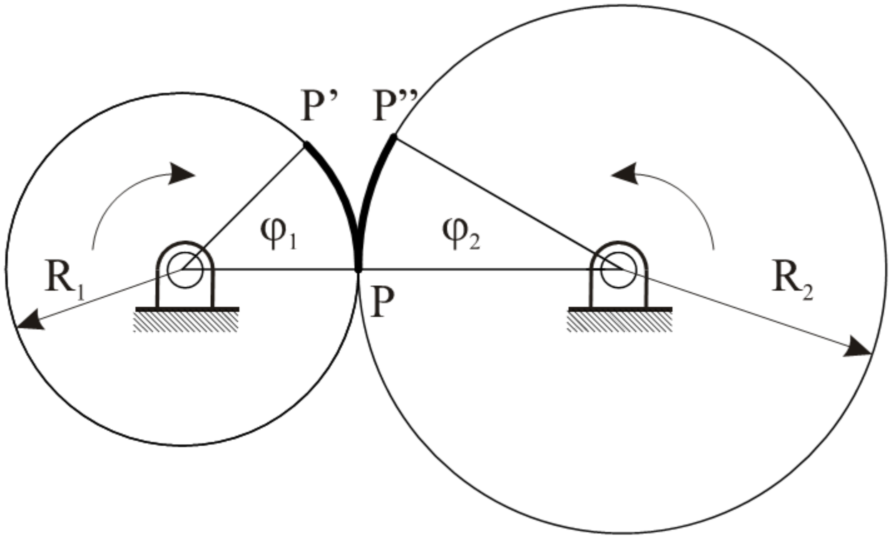


Fig. 2.20